Experimental Verification of a Recursive Method to Calculate Evapotranspiration

Robert J. Lascano
Texas A&M University, 3810 4th Street, Lubbock, TX 79415
(r-lascano@tamu.edu)
and
Steven R. Evett
USDA-ARS, P.O. Drawer 10, Bushland, TX 79012
(srevett@cprl.ars.usda.gov)

ABSTRACT

Recently, a recursive combination method (**RCM**) to calculate potential and crop evapotranspiration (ET) was given by Lascano and Van Bavel (*Agron. J. 2007, 99:585 – 590*). The **RCM** differs from the Penman-Monteith (PM) method, the main difference being, that the assumptions made regarding the temperature and humidity of the evaporating surface in the PM are not necessary when using the **RCM**. Rather, the **RCM** solves for ET by finding the temperature and the humidity by iteration and therefore satisfies the energy balance. We compared values of alfalfa ET measured with a large lysimeter at Bushland, TX, for a range of environmental conditions, to those calculated with the **RCM**. The **RCM** is based on the same physical principles as the PM except for the assumption that air and canopy temperatures are equal in the calculation of vapor pressure vs. air temperature relation. Unlike the PM, the **RCM** uses iteration to find an accurate answer for ET and can be easily be implemented using commercially available mathematical software such as Excel® and Mathcad®. Results for two days show that the **RCM** correctly calculates alfalfa ET and this conclusion is based on the close agreement between measured and calculated hourly values of ET.

INTRODUCTION

In agriculture, information on the amount of water that crops require is necessary to schedule irrigation and to maximize both the efficient use of water resources and crop production. Historically, methods used to estimate water evaporation have been a combination of empirical and theoretical approaches. Reviews of evaporative methods are given by Sibbons (1962), Brutsaert (1982), and more recently by Howell and Evett (2004), and Lascano (2007).

In 1948, *three* seminal papers were published that impacted our understanding of evaporation. *First*, was the paper of Charles W. Thornthwaite (1899 – 1963) (Thornthwaite,

1948) where he introduced and coined the term **potential evapotranspiration** (ET_p) and the concept that ET_p was the maximum rate of water loss by evaporation from the land and depended primarily on atmospheric conditions. A *second* paper was that of Howard L. Penman (1909 – 1984) (Penman, 1948) in which the **combination method** was introduced to ET from open water, bare soil, and grass. It was called the combination method because it combined the energy balance and an aerodynamic or diffusion formula to calculate ET and in doing so eliminated the surface temperature from the relevant equations (e.g., Sibbons, 1962; Milly, 1991). An almost exactly similar solution was obtained independently by Budyko (1951 and 1956) who termed the approach the *complex method* and by Ferguson (1952). The *third* paper was the work of Mikhail I. Budyko (1920–2001) (Budyko, 1948), where he summarized some of his pioneering work on evaporation.

It is clear that the methods proposed by Penman (1948) and by Budyko (1951 and 1956) to calculate evaporation were independent of each other. However, there is a *major* distinction between them in that the assumptions made by Penman (1948) regarding the temperature and the humidity of the evaporating surface are not required with the method proposed by Budyko (1951 and 1956). The method proposed by Budyko was *iterative* and his method consisted of an energy balance equation with two unknowns, ET_p and the surface temperature T_s, and the Goff-Gratch equation (Goff and Gratch, 1945) that relates the saturation humidity at a surface to the temperature at that surface. Starting with an initial value for T_s, the value of both unknowns is found by iteration, resulting in a value of T_s that satisfies the energy balance. An outline of this procedure is given by Budyko (1956, pp. 162 – 163) and by Sellers (1965, pp. 168 – 170). It is of interest to note that the Budyko (1956) publication was used as a graduate-textbook in a climatology class taught by Dr. William D. Sellers while a faculty member at the University of Arizona in Tucson (C.H.M. van Bavel, personal communication). Hereafter, we refer to the procedures based on Penman (1948) as the **E**xplicit **C**ombination **M**ethod (**ECM**) and those based on the iterative procedure first suggested by Budyko (1951 and 1956) as the **R**ecursive **C**ombination **M**ethod (**RCM**). Additional information on the **ECM** and **RCM** is given by Lascano and Van Bavel (2007).

The purpose of this paper is to two-fold. First, we provide a brief historical documentation on the development of **ECM** and **RCM** procedures used to calculate evapotranspiration. Second, using measured hourly values of air and dewpoint temperature, wind-speed, net irradiance, and soil heat flux we calculate ET using the **RCM**, and compare measured and calculated values of alfalfa ET. All measured values were obtained at Bushland, TX and alfalfa ET was measured with large weighing lysimeters (Marek et al., 1988: Howell et al., 1995). The purpose of the second objective was to experimentally verify the **RCM** as proposed by Lascano and Van Bavel (2007).

THEORY

In this section a brief history on the development of the **ECM** and **RCM** procedures is given. First, we start with the Penman (1948) combination equation, which eventually leads to the so-called Penman-Monteith (Monteith, 1965), the FAO-56 (Allen et al., 1998), and the ASCE (2005) procedures, which are all categorized as **ECM**. Second, we present the implementation of the **RCM** procedures based on Budyko (1951 and 1956). Please note that units of terms in most equations are intentionally omitted and if needed the reader should refer to the given references.

Explicit Combination Method (ECM)

Penman (1948) derived an explicit equation for ET_p by combining the energy balance of the evaporating surface with an aerodynamic diffusion equation to describe the flux of water vapor from the surface; thus, the term *combination method* is often used to describe his procedure. The fundamental assumption made by Penman (1948) was to assume that within the range of air and leaf water temperatures the vapor-pressure vs. temperature curve ($\partial e^*/\partial T$) of water might be regarded as a straight line, which he took to be the derivative (tangent) of the vapor pressure curve at the air temperature T_a. This assumption allowed Penman (1948) to eliminate the leaf surface temperature T_s from the equations used to calculate ET_p. Mathematically, the linearity assumption is expressed by an approximation to $\partial e^*/\partial T$ and is given by:

$$\Delta = \frac{e_s^* - e_a^*}{T_s - T_a} \qquad [1]$$

where $\Delta = (de^*/dT)$ is the slope of the saturation vapor pressure curve $e^* = e^*(T)$, at the air temperature T_a, $e^*_a = e^*(T_a)$ is the corresponding saturation vapor pressure, and $e^* = e^*_s(T_s)$ the vapor pressure at the wet surface. Ferguson (1951) solved for ET_p from open water by solving a differential equation without the linearity assumption and derived an identical equation to that given by Penman's (1948) equation (16). A general form of the Penman (1948) equation to describe evaporative flux is given by Howell and Evett (2004):

$$LE = \frac{\left[\Delta\left(R_n - G\right)\right] + \left(\gamma\ LE_a\right)}{\left(\Delta + \gamma\right)} \qquad [2]$$

where LE ($\equiv \lambda E$) is the evaporative latent heat flux (L and/or λ is the latent heat of vaporization); R_n is the net irradiance flux; G is the sensible heat flux into the soil; γ is the psychrometric constant; Δ is the slope of the saturated vapor pressure curve as defined in Eq. [1]; and E_a is the vapor transport flux also known as the aerodynamic evaporative term and empirically defined by Penman (1948) as:

$$E_a = W_f\,(e^* - e_a) \qquad [3]$$

where W_f is called a wind function, e^* is the saturated vapor pressure at mean T_a, and e_a is the mean ambient vapor pressure at a screen height above the ground surface. The evaporative term E_a in Eqs. [2 and 3], for a 24-h period, is expressed using a theoretical adiabatic wind-profile relation that defines the momentum surface aerodynamic resistance r_a, and E_a is given by:

$$E_a = \frac{\frac{\varepsilon\ \rho_a}{P}\ 86{,}400\ (e^* - e_a)}{r_a} \qquad [4]$$

where ε is the mole fraction of water in air (= 0.622), P is the barometric pressure, and ρ_a is the air density (e.g., Businger, 1956; Penman and Long, 1960; Van Bavel, 1966). The r_a for neutral atmospheric conditions is given by Evett (2002):

$$r_a = \frac{\ln\left[\frac{(z_w - d)}{z_{om}}\right]\ln\left[\frac{(z_r - d)}{z_{ov}}\right]}{k^2 U_z} \quad [5]$$

where z_w is the height for wind-speed measurement, z_{0m} is the momentum roughness length, z_r is the measurement height for humidity, z_{0v} is the vapor roughness length, k = 0.41 is von Karman's constant, d is the zero-plane displacement height, and U_z is the wind-speed at screen height z. The aerodynamic crop parameters are empirically estimated, as given by Evett (2002), with the following:

$$d = \frac{2}{3}\ h_c \quad [6]$$

$$z_{om} = 0.123\ h_c \quad [7]$$

$$z_{ov} = 0.1\ z_{om} \quad [8]$$

where h_c is the crop height.

The next step in calculating crop or actual ET was the recognition that an additional resistance to water vapor transport was involved causing $ET_a < ET_p$. For example, Penman (1953) recognized that the transpiration from well-watered vegetation involved a diffusion resistance due to leaf stomata and proposed that the expression for ET_p formulated in 1948 be modified as:

$$LE = \frac{ET_p(\varepsilon + 1)}{\varepsilon + 1 + \frac{r_c}{r_a}} \quad [9]$$

where ε is = Δ/γ, r_c is a canopy resistance term (bulk stomatal resistance), and r_a is the aerodynamic resistance defined in Eq. [5]. Equation [9] is analogous to Penman's (1953) equation (9), where he introduced the empirical concept of a stomatal (S) and a day-length factor (D), is the latter being equivalent to r_c given in Eq. [9]. The day-length factor D was defined as (Penman, 1953):

$$D = \frac{S_0}{24} + \frac{1}{2\pi}\left(\frac{(T_{a,\max} - T_{a,\min})/2}{T_{a,avg} - T_{d,avg}}\right)\sin\left(\frac{S_0\pi}{24}\right) \quad [10]$$

where S_0 is the day-length, $T_{a,max}$ is the daily maximum air temperature, $T_{a,min}$ is the daily air minimum temperature, $T_{a,avg}$ is the daily mean air temperature, and $T_{d,avg}$ is the daily average dewpoint temperature, and all temperatures are measured at a screen height above the ground surface. The stomatal factor S was defined as (Penman, 1953):

$$S = \frac{L_a}{(L_a + L_s)} \qquad [11]$$

where L_a is an empirical function related to the molecular diffusion of water and wind speed at screen height, and $L_s \approx 0.16$, a value calculated for leaves with cylindrical tube stomata, and for a range of stomatal densities and epidermise thicknesses. A general method to calculate canopy resistance from leaf resistance does not exist and has lead to the formulation of theoretical (e.g., Jarvis, 1976) and empirical (e.g., Allen et al., 1989) approaches. Theoretical approaches are not described as they are beyond the scope of this paper.

Empirical equations to estimate bulk surface canopy resistance to water vapor flux based on crop height as a function of leaf area index LAI were given by Allen et al. (1989). For example, for a clipped grass:

$$LAI = 24\ h_c \qquad [12]$$

where h_c is the height of the clipped grass for h_c < 0.15 m. For a non-clipped grass or alfalfa, Allen et al. (1989) proposed:

$$LAI = 1.5\ \ln(h_c) + 5.5 \qquad [13]$$

with a surface or canopy resistance r_c for a reference crop calculated as a function of LAI by:

$$r_c = \frac{100}{(0.5\ LAI)} \qquad [14]$$

The standard reference crop heights adopted by FAO-56 and ASCE are for a grass h_c = 0.12 m and for alfalfa h_c = 0.50 m, which result in r_c = 70 s/m for grass and r_c = 45 s/m for alfalfa.

Penman-Monteith

The resistance values suggested by Allen et al. (1989) were included in various derivations (e.g., Rijtema, 1965; Monteith, 1965) of the Penman (1948) equation and the resulting equation is known as the **Penman-Monteith** equation, which for daily values of LE is given by:

$$LE = \frac{\Delta\left(R_n - G\right) + \dfrac{86{,}400 \rho_a C_p \left(e_s^* - e_a\right)}{r_a}}{\Delta + \gamma\left(1 + \dfrac{r_c}{r_a}\right)} \qquad [15]$$

where ρ_a is the air density, C_p is the specific heat of dry air, e^*_s is the mean saturated vapor pressure, e_a is the mean daily ambient vapor pressure, r_c is the canopy surface resistance, and r_a is the bulk surface aerodynamic resistance for water vapor. This equation is known as the ASAE Penman-Monteith equation (Jensen et al., 1990).

The ASCE-Standardized Reference Evapotranspiration Equation

The equation adopted and recommended by FAO-56 (Allen et al., 1998) and by ASCE (2005) to calculate crop evapotranspiration is based on the Penman-Monteith equation as given by Eq. [15]. Furthermore, to simplify and as an attempt to standardize the calculation ASCE adopted what is now termed as a Standardized Reference Evapotranspiration Equation, ET_{sz}, which for calculation of daily values is given by:

$$ET_{sz} = \frac{0.408\Delta(R_n - G) + \gamma \dfrac{C_n}{T+273} u_2 (e_s - e_a)}{\Delta + \gamma(1 + C_d u_2)} \qquad [16]$$

where ET_{sz} is the standardized reference crop ET for short (ET_{os}) or for tall crop surfaces (ET_{rs}), both in mm/d; R_n is the calculated net irradiance at the crop surface; G is the soil heat flux density at the soil surface, both terms in MJ/(m^2 d); T is the measured mean daily air temperature (°C); u_2 is the mean daily wind speed (m/s) measured at a screen height = 2 m; e_s is the saturation vapor pressure (kPa) calculated for daily time steps as the average of the saturation vapor pressure at maximum and at minimum air temperature; e_a is the mean actual vapor pressure (kPa); Δ is the slope of the saturation vapor-pressure temperature curve (kPa/°C), γ is the psychrometric constant (kPa/°C); C_n [K mm s^3/(Mg d)] is the numerator constant; and, C_d (s/m) is the denominator constant and both change with crop reference type and calculation time-step. The units for the coefficient 0.408 are m^2 mm/MJ. The screen-height for the measurement of T, e_s and e_a can vary between 1.5 and 2.5 m. Details on how to calculate R_n and G for daily estimates of ET_{sz} are given by Allen et al. (1998) and by ASCE (2005).

In practice, Eq. [16] is commonly used to calculate a daily reference ET for either a grass or an alfalfa crop, using values of short-wave irradiance, air and dewpoint temperature, and wind-speed, commonly measured at a screen height of 2.0 m above the ground surface. It is suggested that weather inputs used be based on hourly measurements of the weather variables (e.g., ASCE, 2005). This procedure is commonly used by regional weather networks, e.g., the Texas High Plains ET Network (http://txhighplainset.tamu.edu/), dedicated to providing information for the irrigation management of crops. The procedure is to calculate daily values of reference ET for a short grass using Eq. [16] and to multiply this value by crop-specific coefficients, thus providing a daily estimate of crop ET. In the Texas High Plains, this procedure is used to estimate the daily water requirements of crops such as, cotton, corn, soybean and wheat. This general method of using a crop reference ET in combination with crop coefficients to estimate crop ET, was termed the *engineering-approach* (Lascano, 2000) and was first suggested by Jensen (1968). For additional information see Lascano (2007).

Linearity Assumption

In Eq. [1], as T_s departs from T_a the error in the value of ET increases and this occurs under environmental conditions that are conducive to low and high rates of evaporation and high levels of solar irradiance (e.g., Sellers, 1965; Milly, 1991). This is important because conditions of high evaporation and solar irradiance are normally associated with arid and semi-arid environments where crop irrigation is normally practiced. Furthermore, the validity of the assumption of using a linear expansion of the curve of saturation vapor pressure curve vs. air temperature as introduced by Penman (1948) has been questioned by others (Sellers, 1965; Tracy et al., 1984; Paw U and Gao, 1988; McArthur, 1990 and 1992; Milly, 1991; and, Paw U, 1992). These authors suggested several approaches to eliminate the linearity assumption and thus minimize errors when calculating ET.

Paw U and Gao (1988) used a second-order Taylor expansion series of the approximation given by Eq. [1], i.e.,

$$e_s(T_a) = e_s(T_a) + \Delta(T_s - T_a) + \frac{1}{2}\frac{d^2 e_s}{dT^2}(T_s - T_a)^2 \qquad [17]$$

where the surface temperature T_s is eliminated (similar to Penman, 1948), yielding a quadratic equation for latent heat flux density (LE):

$$aLE^2 + bLE + c = 0 \qquad [18]$$

where coefficients a, b, and c are related to environmental parameters. This equation should have less error than the Penman (1948) equation because the saturation vapor pressure function is approximated by a quadratic curve instead of a straight line. Another solution to LE given by Paw U and Gao (1988) involved a quartic equation, expressing the saturation vapor pressure function by the approximation:

$$e_s(T_s) = \zeta + \alpha T_s + \beta T_s^2 + \psi T_s^3 + \mu T_s^4 \qquad [19]$$

where by algebraic manipulation and substitution into the energy balance equations T_s is eliminated and thus yielding a quartic equation to solve for LE:

$$kLE^4 + a^{'}LE^3 + b^{'}LE^2 + c^{'}LE + d^{'} = 0 \qquad [20]$$

where the coefficients k, $a^{'}$, $b^{'}$, $c^{'}$, and $d^{'}$ are related to parameters of the energy balance. The solution to Eq. [20] is complex and is given by Paw U and Gao (1988) in their appendix. Nevertheless, this solution still represents an approximation, although the error should be less than when a linear approximation is used. Additional information on Eqs. [18, 19, and 20] is given by Paw U (1992).

Milly (1991) went a step further and introduced a higher-order Taylor series to evaluate the saturation vapor pressure function using the expression:

$$T_s = T_a + \sum_{m=1}^{\infty} \frac{1}{m!}\left[\frac{d^m T^*}{de^m}\right]_a (e^*(T_s) - e^*(T_a))^m \qquad [21]$$

where T_a is the air temperature at an arbitrary level in the atmosphere, T_s is the air temperature near (adjacent) the evaporating surface, and m = 1,2,3 … ,. Specifically, the subscript a refers to conditions at an arbitrary level in the atmosphere above the surface. Again, by algebraic manipulation and substitution into general energy balance equations Milly (1991) derived a solution to calculate ET_a, which is given by his equation (23), although a simplification is given by his equation (25), which follows:

$$ET_a = \frac{\Delta_a (R_n - G)/L}{\Delta_a + \gamma^*}\left[1 + A + 0.44\frac{(1 - A\Delta_a/\gamma^*)^2}{A(1+\Delta_a/\gamma^*)^2}\sigma\right] \qquad [22]$$

where ET_a is the evaporation rate; R_n is the net irradiance; G is the soil heat flux; L is the latent heat of vaporization, $\gamma^* = \gamma(r_a + r_c)/r_{ah}$, where γ is the psychrometric constant, r_a is the aerodynamic resistance to water vapor transport, r_c is the bulk canopy (stomatal) resistance, and r_{ah} is the aerodynamic resistance to heat transport; $\sigma = (d_a - d_{st})/e^*(T_a)$, where d_a is the vapor pressure deficit of the air, d_{st} is the vapor pressure deficit within stomatal cavities, Δ_a is the slope of the saturation vapor pressure curve; and, A is defined by:

$$A = \frac{\rho\, C_p (d_a - d_{st})}{\Delta_a (R_n - G) r_{ah}} \qquad [23]$$

where ρ is the air density, C_p is the specific heat at a constant pressure, and other terms are as previously defined. Milly (1991) through various manipulations also derived the quadratic equation, i.e., Eq. [19], given by Paw U and Gao (1988), leading to Milly's (1991) equation (29).

Another contribution of Milly (1991) was the derivation of a relative error term ε_r in evaporation rate ET_a calculated with **ECM** equations that use the linear assumption introduced by Penman (1948). The error ε_r is given by:

$$\varepsilon_r = -\frac{0.44(1 - A\Delta_a/\gamma^*)^2\,\sigma}{A(1+A)(1+\Delta_a/\gamma^*)^2} \qquad [24]$$

showing that ε_r is always non-positive, i.e., the **ECM** to calculate ET_a can only yield smaller values of evaporation rate than does the **RCM**, but can never yield larger values than the **RCM**, but can never yield larger values than the **RCM**, all other factors being equal. In addition, it shows that ε_r goes to zero when the ratio A of the so-called 'wind term' to the 'radiation term' is equal to γ^*/Δ_a. This condition only occurs when LE ≡ (R_n – G), i.e., the sensible heat flux is zero and therefore $T_a \equiv T_s$.

The solutions to LE given by Paw U and Gao (1988) and Milly (1991) represent an improvement over the solutions given by the **ECM**, and used by FAO-56 (Allen et al., 1998) and ASCE (2005). Milly (1991) refers to these types of equations as *first-order combination equations*. However, the solutions are complex and convergence is not always assured. Tracy et al. (1984), McArthur (1992), and Milly (1991) stated that only by iteration can complete accuracy be obtained. Iterative methods are not new and have been used to calculate water evaporation from the soil and plant using energy and water balance simulation models (e.g., Lascano and Van Bavel, 1983 and 1986; Lascano, et al., 1987). Bristow (1987) used a Newton iterative procedure to find the surface temperature in solving the energy balance equation. However, none of these iterative techniques has been applied to provide general calculations of ET. Current data-loggers used with weather stations that measure the necessary weather input parameters have the necessary storage and processing capabilities.

Recursive Combination Method (RCM)

In addition to his earlier work (Budyko, 1951), Budyko (1956, pp. 162 – 163) suggested without any assumptions, an energy balance equation with two unknowns, ET and the surface temperature T_s, and the Goff and Gratch (1945) equations that relate the saturation humidity at the surface to T_s. The values of both unknowns (ET and T_s) are found by iteration starting with an initial value for T_s that satisfies the energy balance. Additional information is given by Lascano and Van Bavel (2007).

Lascano and Van Bavel (2007) used the mathematical software Mathcad[1]® v. 13 (Mathsoft Engineering & Education, Inc., Cambridge, MA) and Microsoft® Excel 2002, for the iterative solution of actual and potential ET. Mathcad® v. 13 uses the Secant or Muller method in the solution. In Mathcad® syntax the iterative calculation of T_s is given by Eq. [25], below:

$$T_s := root\left[\left[\left[0.80\times R_g - 5.67\times 10^{-8}\times\left(T_s+273.2\right)^4 + R_l\right] + \frac{\left(T_a-T_s\right)\times\rho_d\times c_p}{r_a} - \frac{\left(1.323\times\frac{\exp\left(17.269\times\frac{T_s}{T_s+237.0}\right)}{T_s+273.2} - 1.323\times\frac{\exp\left(17.269\times\frac{T_d}{T_d+237.0}\right)}{T_a+273.2}\right)}{r_a+r_c}\times\lambda\right]\times\frac{r_a}{c_p\times\rho_d},\ T_s\right] \quad [25]$$

[1] The mention of trade names of commercial products in this article is solely for the purpose of providing specific information and does not imply recommendation or endorsement by the U.S. Department of Agriculture.

where *root* is a built-in function given in Mathcad® where an initial value of T_s is given, e.g., T_s =10.0. In Eq. [25] inside the brackets is the energy balance of the crop, where the first term is the radiative component, the second term is the sensible heat flux, and the third term is the latent heat flux. All terms in Eq. [25] have been previously defined, except R_g the incoming short-wave irradiance, R_l the sky long-wave irradiance, and T_d the dewpoint air temperature at screen height. Once the implicit value of T_s is found, the latent and sensible heat fluxes are known as functions of T_s. We have also used *solver* (Excel® 2002) and compared results of T_s and ET to the solutions obtained with Mathcad® and the results from both solutions were identical.

METHODS

Experimental Data

Experimental weather and alfalfa ET data were gathered at the USDA-ARS Conservation and Production Research Laboratory, Bushland, TX (35° 11′ N, 102° 06′ W, 1170 m elevation above MSL) on a Pullman fine, mixed, superactive, thermic Torrertic Paleustoll soil. Half-hourly values of air (T_a) and dew-point (T_d) temperature, net (R_n) and incoming short-wave (R_g) irradiance, soil heat flux (G), wind-speed (U_z), and surface radiometric temperature (T_s) for 27 days with no rain in 1999 were selected and used as input data to calculate hourly values of ET_p using **RCM**. Half-hourly values of alfalfa ET were measured with large weighing lysimeters. Additional input data was the measured alfalfa height for the 27 selected days. Alfalfa variety Pioneer 5454 was seeded at a rate of 28 kg/ha on 13 – 14 Sep 1995, with a grain drill on 0.2-m spacing. Alfalfa was irrigated and fertilized to produce a reference ET vegetative surface (well-watered and without limitation of fertilizer or other inputs or management). A general description of the sensors and methods used to measure the above variables is given by Evett et al. (2000).

Lysimeter mass was measured every 0.5-h with 0.05-mm precision (Dusek et al., 1987). Weather variables were measured every 6-s and reported on 0.5-h averages. Over the lysimeter, R_n was measured with REBS net radiometers (Q*5.5, Seattle, WA), G was measured with four heat flux plates (REBS, HFT-1, Seattle, WA) buried 0.05-m below the surface with averaging thermocouples at 0.02- and 0.04-m above each plate. Also, T_s was measured with infrared thermometers (Everest, Model 4000, Fullerton, CA). Air and dew-point temperature, and wind-speed were measured at a screen height of 2.0 m in a nearby grass weather station using standard procedures as given by Evett (2002).

Procedures

For the purpose of this paper, only 3 days of measurement in 1999 were selected and used to validate the **RCM** of crop ET. A description of the procedure used in our calculations follows.

1. Crop ET was calculated using 0.5-h weather data using **RCM** of Lascano and Van Bavel (2007) as given by Eq. [25] and using the aerodynamic resistance (r_a) defined by Eq. [5]. Using the Mathcad® software (v. 14, Parametric Technology Corporation, Needham,

MA), canopy resistance (r_c) was defined as a *range-variable* and the measured values of crop ET and surface radiometric temperature (T_s) were used to estimate hourly values of r_c during daylight hours when stomata are fully open. In this procedure T_s, sensible heat flux, and ET were each calculated for values of r_c ranging between 10 and 100 s/m in 10 s/m increments. Data for Day of Year (DOY) 185 (4 July 1999), were used for this purpose.

2. Using the values of canopy resistance (r_c), as calculated with the previously described procedure, hourly values of alfalfa ET were calculated for two days (DOY 182 and 183) using as input measured values of weather variables, again using the **RCM**. The values of r_c calculated by this procedure are the values that satisfy the measured alfalfa ET values and corresponding radiometric surface temperatures.

RESULTS AND DISCUSSION

Calculation of Canopy Resistance (r_c)

The concept of using measured values of crop ET obtained with lysimeters to estimate canopy resistance (r_c) from a well-watered crop that is actively transpiring was first used by Ehrler and Van Bavel (1967), and Van Bavel and Ehrler (1968) on a sorghum crop. Using this procedure along with the **RCM** to calculate ET, the r_c was defined as a *range-variable* (10 – 100 s/m) and at 13:00 h on DOY = 185, the measured value of alfalfa ET was 1.026 mm, which yielded a value of r_c = 31.5 s/m (Fig. 1).

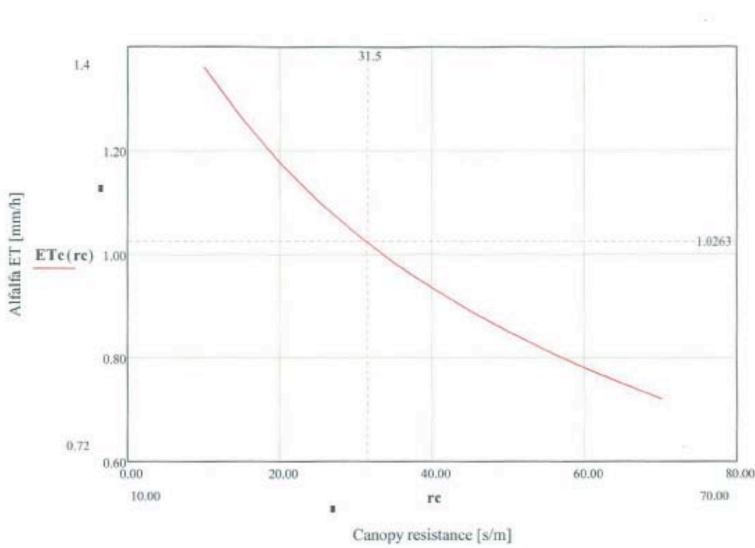

Figure 1. Calculated alfalfa-ET using **RCM** as a function of canopy resistance, defined as a *range-variable* in Mathcad® v. 14. The lysimetric measurement of alfalfa ET for this time-period was 1.026 mm, which yielded an r_c value of 31.5 s/m.

A second procedure that can also be used to obtain an estimate of r_c is to use the measurement of radiometric surface temperature (T_s) in a similar way as the measurement of crop ET (Fig. 1). Again, r_c was defined as a *range-variable* (10 – 100 s/m) in Mathcad® v. 14. At 13:00 h on DOY = 185, a measured T_s = 27.5 °C gives a calculated r_c = 33.5 s/m (Fig. 2).

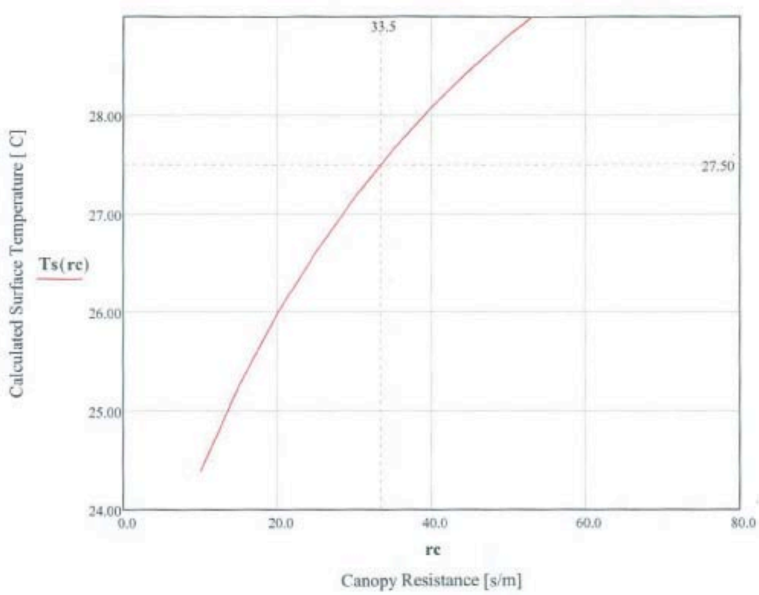

Figure 2. Calculated surface temperature (T_s) using **RCM** as a function of canopy resistance, defined as a *range-variable* in Mathcad® v. 14. The surface temperature measurement over alfalfa for this time-period was 27.5 °C, which yields an r_c value of 33.5 s/m.

A comparison between calculated values of canopy resistance (r_c) for DOY 185, 1999, between 8:00 – 19:00 h, for the two procedures used is shown in Fig. 3. The average between 10:00 – 17:00 h, for r_c from lysimetric measurements was 32 s/m (standard deviation = 2 s/m), and derived from surface radiometric temperature, it was 41 s/m (standard deviation = 8 s/m). Between 12:00 – 14:00 h, the calculated values of r_c from both lysimetric and radiometric surface temperatures are similar ~ 33 s/m. Therefore, we selected r_c = 33 s/m as the canopy resistance value to calculate hourly alfalfa ET using the **RCM** for DOY 182 and 183, and these values were compared to measured values of alfalfa-ET obtained with the lysimeter. The value of r_c reported by Ehrler and Van Bavel (1967) for three midday hourly values for sorghum was 28 s/m, i.e., 18% lower than that measured for alfalfa and shown in Fig. 3.

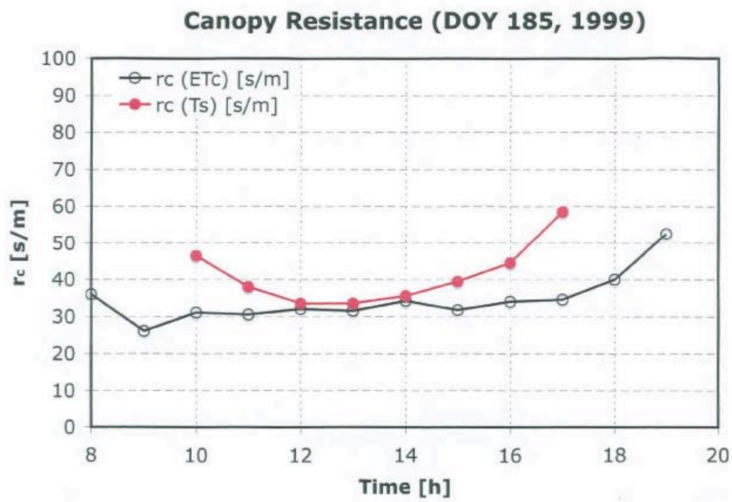

Figure 3. Calculated values of canopy resistance (r_c) from radiometric surface temperature T_s (●) and from lysimetric measurements ET_c (O) for DOY 185, 1999.

Weather-data for DOY 182

To illustrate the type of hourly input data used in our examples to calculate ET, we selected DOY 182, 1999. The hourly measured variables of air temperature (T_a), dewpoint temperature (T_d) and radiometric alfalfa canopy surface temperature (T_{can}) are shown in Fig. 4. The average daily T_a was 24.2 °C and average T_d was 18.0 °C. The corresponding measured hourly wind speed is shown in Fig. 5 and the daily average was 4.6 m/s. Hourly measured fluxes of net irradiance (R_n), soil heat flux (G), sensible heat flux (SH) and latent heat flux (LE) are shown in Fig. 6. The daily integrated fluxes were R_n = 17.14 MJ/m^2, SH = 14.51 MJ/m^2, LE = -32.04 MJ/m^2, and G = 0.39 MJ/m^2.

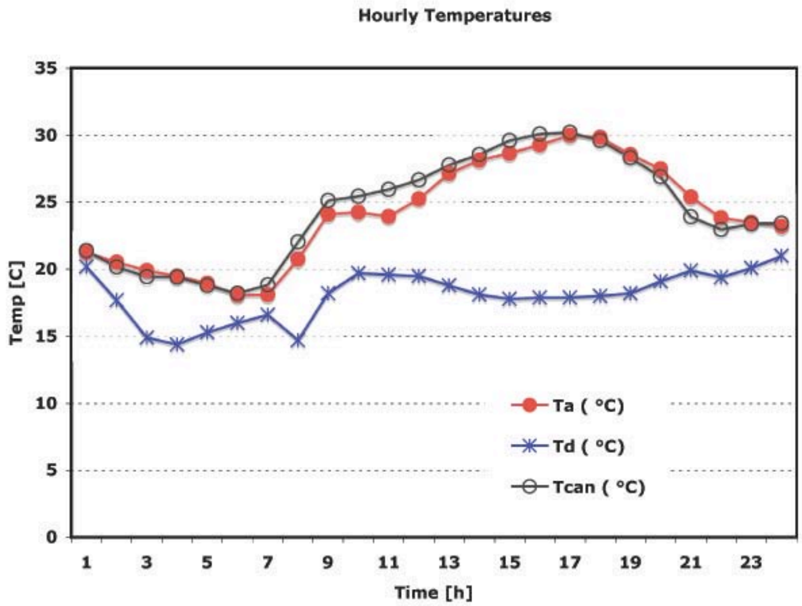

Figure 4. Hourly measured values of air temperature (T_a), dewpoint temperature (T_d), and radiometric alfalfa canopy temperature (T_{can}) on DOY 182, in Bushland, TX.

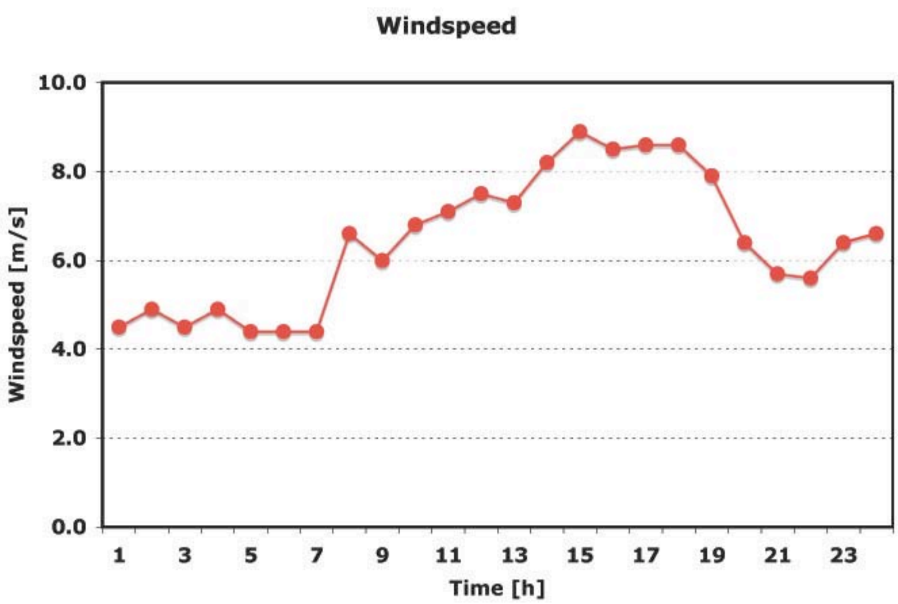

Figure 5. Hourly measured values of wind-speed for DOY 182, 1999 in Bushland, TX.

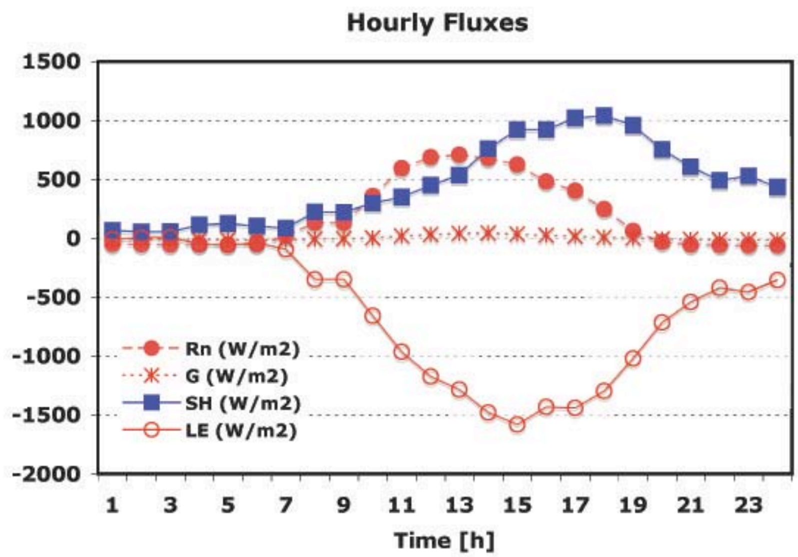

Figure 6. Hourly fluxes of net irradiance (R_n), soil heat (G), sensible heat (SH), and latent heat (LE) for DOY 182, in Bushland, TX.

The daily calculated value of potential ET_p obtained using the input weather data shown in Figs. 4 and 5, and using the **RCM** gave a total of 13.1 mm for DOY 182, 1999.

Comparison of Measured and Calculated Alfalfa-ET

Comparison of measured and calculated hourly values of alfalfa ET, between 8:00 – 18:00 h, assuming a constant values of r_c = 33 s/m throughout the day for the two selected days showed close agreement (Figs. 7 and 8). Furthermore, linear regression analysis (Fig. 9) indicated that the slope was not significantly different from 1.0 and the intercept was not significantly different than 0.0, with an r^2 = 0.98. From this comparison, we can conclude that the recursive combination method (**RCM**) first proposed by Budyko (1951 and 1956) and formulated by Lascano and Van Bavel (2007) is workable. The **RCM** is based on the same physical principles of the Penman-Monteith solution to ET, but uses iteration to find an accurate answer. It remains to be seen if the **RCM** compares well with the **ECM** for daily as well as seasonal estimation of alfalfa reference ET, which is work that we have in progress.

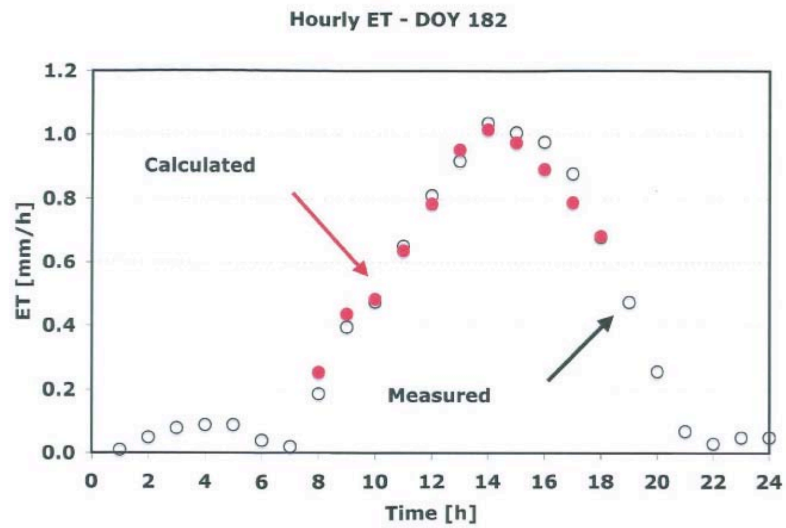

Figure 7. Hourly values of alfalfa ET measured with a lysimeter and calculated with **RCM** assuming r_c = 33 s/m, for DOY 182, 1999. Calculated values of ET were only done for daylight hours, i.e., 8:00 – 18:00 h.

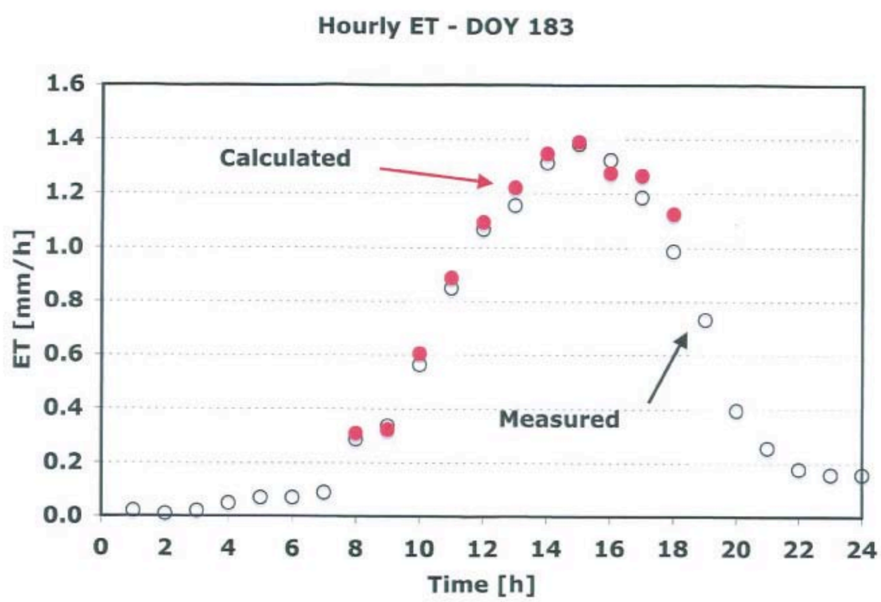

Figure 8. Hourly values of alfalfa ET measured with a lysimeter and calculated with **RCM** assuming r_c = 33 s/m, for DOY 183, 1999. Calculated values of ET were only done for daylight hours, i.e., 8:00 – 18:00 h.

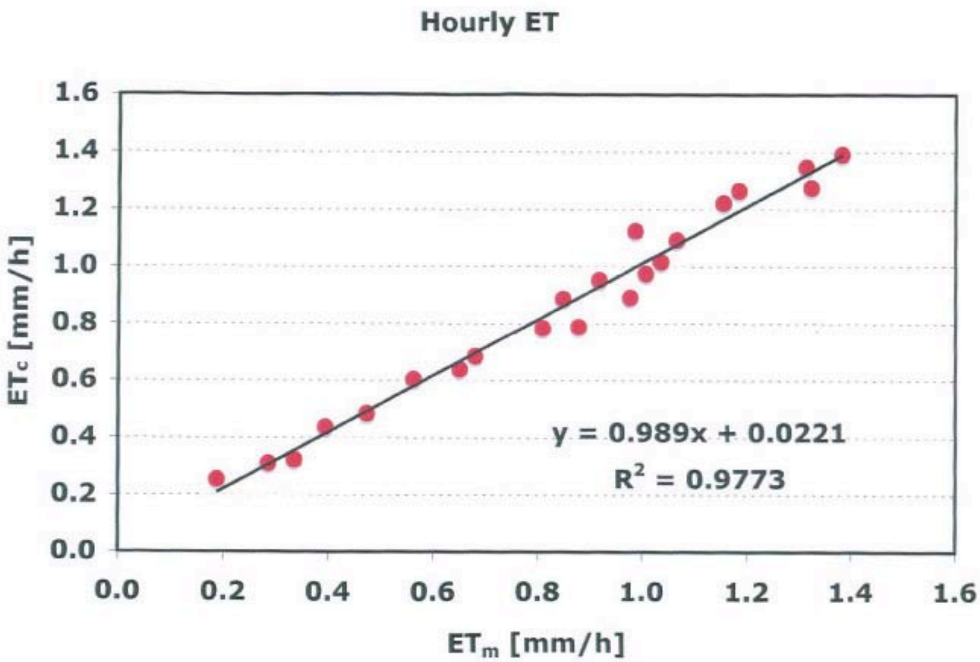

Figure 9. Linear regression between calculated values of ET (ET_c) and measured values of ET (ET_m) for the two days shown in Figs. 7 and 8.

REFERENCES

Allen, R.G., M.E. Jensen, J.L. Wright, and R.D. Burman. 1989. Operational estimates of evapotranspiration. Agron. J. 81:650–662.

Allen, R.G., S. Pereira, D. Raes, and M. Smith. 1998. Crop Evapotranspiration: Guidelines for Computing Water Requirements. Food and Agriculture Organization (FAO), Irrigation and Drainage Paper 56, Rome, 300 pp.

ASCE, 2005. The ASCE Standardized Reference Evapotranspiration Equation. Prepared by Task Committee on Standardization of Reference Evapotranspiration of the Environmental and Water Resources Institute. January, 2005. Environmental and Water Resources Institute of the American Society of Civil Engineers (Available on-line http://www.kimberly.uidaho.edu/water/asceewri/ascestzdetmain2005.pdf). Last accessed on 19 Sept. 2006.

Bristow, K.L. 1987. On solving the surface energy balance equation for surface temperature. Agric. For. Meteorol. 39:49–54.

Brutsaert, W. 1982. Evaporation into the Atmosphere. Theory, History, and Applications. Kluwer Academic Publishers, Dordrecht, The Netherlands, 299 pp.

Budyko, M.I. 1948. Evaporation under Natural Conditions, GIMIZ, Leningrad. (Translated from Russian, Published by the National Science Foundation, Washington, D.C., and the Dept. of the Interior Israel Program for Scientific Translations, Jerusalem, 1963), 130 pp.

Budyko, M.I. 1951. The effect of ameliorative measures on evaporation. Izvestiia of the Academy of Sciences S.S.S.R., Ser Geograficheskaia, No. 1, 16–35. (Cited from Sibbons, 1962 and Milly, 1991).

Budyko, M.I. 1956. The Heat Balance of the Earth's Surface. (English translation, N.A. Stepanova, Office of Technical Services, PB 131692, U.S. Department of Commerce, Washington, D.C., 1958), 259 pp.

Businger, J.A. 1956. Some remarks on Penman's equations for evapotranspiration. Netherlands J. Agric. Sci. 4:77–80.

Dusek, D.A., T.A. Howell, A.D. Schneider, and K.S. Copeland. 1987. Bushland weighing lysimeters data acquisition systems for evapotranspiration research. ASAE Paper No. 87-2506.

Ehrler, W.L., and C.H.M. van Bavel. 1967. Sorghum foliar response to changes in soil water content. Agron. J. 59:243–246.

Evett, S.R. 2002. Water and Energy Balances at Soil-Plant-Atmosphere Interfaces. Chapter 5. *In:* Soil Physics Companion, A.W. Warrick (ed). CRC-Press, Boca Raton, FL, pp. 127 – 188.

Evett, S.R., T.A. Howell, R.W. Todd, A.D. Schneider, and J.A. Tolk. 2000. Alfalfa reference ET measurement and prediction. pp. 266-272, *In*: R.G. Evan, B.L. Benham, and T.P. Trooien (ed). Proceedings of the 4th Decennial National Irrigation Symposium, Nov 14-16, Phoenix, AZ.

Ferguson, J. 1951. The rate of natural evaporation from shallow ponds. Australian J. Scientific Res. Series A, Physical Sciences. 5:315–330.

Goff, J.A., and S. Gratch. 1945. Thermodynamic properties of moist air. Trans. Am. Soc. Heating and Ventilating Eng. 51:125–164.

Howell, T.A., and S.R. Evett. 2004. The Penman-Monteith Method. Section. *In* Evapotranspiration Determination of Consumptive Use in Water Rights Proceedings. Continuing Legal Education in Colorado, Inc. Denver, CO. 14pp. (Available on-line http://www.cprl.ars.usda.gov/wmru/pdfs/PM%20COLO%20Bar%202004%20corrected%209apr04.pdf).

Howell, T.A., A.D. Schneider, D.A., Dusek, T.H. Marek, and J.L. Steiner. 1995. Calibration and scale performance of Bushland weighing lysimeters. Trans. ASAE. 38:1019–1024.

Jarvis, P.G. 1976. The interpretation of leaf water potential and stomatal conductance found in canopies in the field. Phil. Trans. R. Soc. London, Ser. B, 273:593–610.

Jensen, M.E. 1968. Water consumption by agricultural plants. p. 1–22. *In* T.T. Kozlowski (ed.) Water deficits in plant growth. Vol. II. Academic Press, San Diego, CA.

Jensen, M.E., R.D. Burman, and R.G. Allen (eds). 1990. Evaporation and Irrigation Water Requirements. ASCE Manuals and Reports on Eng. Practices No. 70, Am. Soc. Civil Eng., NY, 360pp.

Lascano, R.J. 2000. A general system used to measure and calculate daily crop water use. Agron. J. 92:821–832.

Lascano, R.J. 2007. The Soil-Plant Atmosphere and Monitoring Technology, Chapter 3, pp. 85 – 116. *In* R.J. Lascano and R.E. Sojka (eds). Irrigation of Agricultural Crops., Agron. Monograph No. 30 (2nd edition), ASA, CSSA, SSSA, Madison, WI.

Lascano, R.J., and C.H.M. van Bavel. 1983. Experimental verification of a model to predict soil moisture and temperature profiles. Soil Sci. Soc. Am. J. 47:441–448.

Lascano, R.J., and C.H.M. van Bavel. 1986. Simulation and measurement of evaporation from a bare soil. Soil Sci. Soc. Am. J. 50:1127–1132.

Lascano, R.J., and C.H.M. van Bavel. 2007. Explicit and recursive calculation of potential and actual evapotranspiration. Agron. J. 99: 585 – 590.

Lascano, R.J., CH.M. van Bavel, J.L. Hatfield, and D.R. Upchurch. 1987. Energy and water balance of a sparse crop: Simulated and measured soil and crop transpiration. Soil Sci. Soc. Am. J. 51:1113–1121.

Marek, T.H., A.D. Schneider, T.A. Howell, and L.L. Ebeling. 1988. Design and construction of large weighing monolithic lysimeters. Trans. ASAE 31:477–484.

Monteith, J.L. 1965. Evaporation and environment, pp. 205–234. *In* G.E. Fogg (ed). Symposium of the Society for Experimental Biology, The State and Movement of Water in Living Organisms, Vol. 19, Academic Press, Inc., N.Y.

McArthur, A.J. 1990. An accurate solution to the Penman equation. Agric. For. Meteorol. 51:87–92.

McArthur, A.J. 1992. The Penman for equations and the value of Delta: a small difference of opinion or a matter of fact? Agric. For. Meteorol. 57:305–308.

Milly, P.C.D. 1991. A refinement on the combination equations for evaporation. Surveys in Geophysics 12:145–154.

Paw U, K.T. 1992. A discussion of the Penman form equations and comparisons of some equations to estimate latent energy flux density. Agric. For. Meteorol. 57:297–304.

Paw U, K.T., and W. Gao. 1988. Applications of solutions to non-linear energy budget equations. Agric. For. Meteorol. 43: 121–145.

Penman, H.L. 1948. Natural evaporation from open water, bare soil, and grass. Proc. Roy. Soc. London. A193:120–146.

Penman, H.L. 1953. The physical bases of irrigation control. Report 13th Int. Hort. Congress: 1952, 2:913–924.

Penman, H.L., and I.F. Long. 1960. Weather in wheat. Quarterly J. Roy. Meteorological Soc. 86:16–50.

Rijtema, P.E. 1965. Analysis of actual evapotranspiration. Agric. Res. Rep. No. 69, Centre for Agric. Publ. and Doc. Wageningen, The Netherlands, 111pp.

Sellers, W.D. 1965. Physical Climatology. The University of Chicago Press, Chicago, 272 pp.

Sibbons, J.L.H. 1962. A contribution to the study of potential evapotranspiration. Geografiska Annaler. 44:279–292.

Thornthwaite, C.W. 1948. An approach toward a rational classification of climate. Geogr. Rev. 38:55–94.

Tracy, C.R., F.H. van Berkum, J.S. Tsuji, R.D. Stevenson, J.A. Nelson, B.M. Barnes, and R.B. Huey. 1984. Errors resulting from linear approximations in energy balance equations. J. Therm. Biol. 9:261–264.

Van Bavel, C.H.M. 1966. Potential evaporation: the combination concept and its experimental verification. Water Resour. Res. 2:455–467.

Van Bavel, C.H.M., and W.L. Ehrler. 1968. Water loss from a sorghum field and stomatal control. Agron. J. 60:84–86.